NAG Toolbox for MATLAB

f04ac

1 Purpose

f04ac calculates the approximate solution of a set of real symmetric positive-definite band equations with multiple right-hand sides, using a Cholesky factorization.

2 Syntax

```
[c, rl, ifail] = f04ac(a, b, n, m, 'ir', ir, 'm1', m1)
```

3 Description

Given a set of real linear equations $AX = B$, where A is a symmetric positive-definite band matrix, the function computes a Cholesky factorization of A as $A = LL^{\mathrm{T}}$, where L is a lower triangular band matrix. The columns x of the solution X are found by forward and backward substitution in $Ly = b$ and $L^{\mathrm{T}}x = y$, where b is a column of the right-hand side matrix B.

4 References

Wilkinson J H and Reinsch C 1971 *Handbook for Automatic Computation II, Linear Algebra* Springer–Verlag

5 Parameters

5.1 Compulsory Input Parameters

1: **a(lda,m1) – double array**

lda, the first dimension of the array, must be at least **n**.

The lower triangle of the n by n positive-definite symmetric band matrix A, with the diagonal of the matrix stored in the $(m+1)$th column of the array, and the m subdiagonals within the band stored in the first m columns of the array. Each row of the matrix is stored in the corresponding row of the array. For example, if $n = 5$ and $m = 2$, the storage scheme is:

$$\begin{pmatrix} * & * & a_{11} \\ * & a_{21} & a_{22} \\ a_{31} & a_{32} & a_{33} \\ a_{42} & a_{43} & a_{44} \\ a_{53} & a_{54} & a_{55} \end{pmatrix}.$$

The elements in the top left corner of the array are not used. The following code may be used to assign elements within the band of the lower triangle of the matrix to the correct elements of the array:

```
for i=1:n
  for j=max(1,i-m):i
    a(i,j-i+m+1) = matrix(i,j);
  end
end
```

2: **b(ldb,ir) – double array**

ldb, the first dimension of the array, must be at least **n**.

The n by r right-hand side matrix B. See also Section 8.

3: **n – int32 scalar**

n, the order of the matrix A.

4: **m – int32 scalar**

m, the number of subdiagonals within the band of A.

5.2 Optional Input Parameters

1: **ir – int32 scalar**

Default: The dimension of the arrays **b**, **c**. (An error is raised if these dimensions are not equal.)

r, the number of right-hand sides.

2: **m1 – int32 scalar**

Default: The dimension of the arrays **a**, **rl**. (An error is raised if these dimensions are not equal.)

the value $m + 1$.

5.3 Input Parameters Omitted from the MATLAB Interface

lda, ldb, ldc, ldrl

5.4 Output Parameters

1: **c(ldc,ir) – double array**

The n by r solution matrix X. See also Section 8.

2: **rl(ldrl,m1) – double array**

The lower triangular band matrix L stored in the same form as A, except that the reciprocals of the diagonal elements are stored instead of the elements themselves.

3: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

A is not positive-definite, possibly due to rounding errors.

7 Accuracy

The accuracy of the computed solutions depend on the conditioning of the original matrix. For a detailed error analysis see page 54 of Wilkinson and Reinsch 1971.

8 Further Comments

The time taken by f04ac is approximately proportional to $n(m + 1)^2$.

This function should only be used when $m \ll n$ since as m approaches n, it becomes less efficient to take advantage of the band form.

9 Example

```
a = [0, 0, 5;
     0, -4, 6;
     1, -4, 6;
     1, -4, 6;
     1, -4, 6;
     1, -4, 6;
     1, -4, 5];
b = [0;
     0;
     0;
     1;
     0;
     0;
     0];
n = int32(7);
m = int32(2);
[c, rl, ifail] = f04ac(a, b, n, m)
```

```
c =
    4.0000
    7.5000
   10.0000
   11.0000
   10.0000
    7.5000
    4.0000
rl =
         0         0    0.4472
         0   -1.7889    0.5976
    0.4472   -1.9124    0.6831
    0.5976   -1.9518    0.7385
    0.6831   -1.9695    0.7774
    0.7385   -1.9789    0.8062
    0.7774   -1.9846    1.4790
ifail =
           0
```